

# A Creep Model with Different Properties Under Tension and Compression – Applications to Refractory Materials

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## Abstract

Refractories are materials designed to work at high temperatures, and are applied in steel making, cement making, aerospace engineering, and other fields where a combination of chemical and mechanical stability is paramount. Due to such high temperature applications, creep strains play an important role in the mechanical performance of refractories, that often present an asymmetric behavior, i.e., different creep strain rates under tension and compression. The aim of this work is to propose an asymmetric creep model that can be used to simulate the time-dependent non-linear primary and secondary creep behavior of refractories at high temperature. The proposed model uses a split of the stress tensor into positive and negative parts and further calculation of the contribution of each stress sign to the overall strain rate using a weighted average over the equivalent stresses. An experimental procedure is proposed in order to identify the tensile and compressive parameters of an alumina-spinel refractory used in steel ladle linings, using a Brazilian test and a subset-based Digital Image Correlation (DIC) technique, for a temperature of 1300 °C.

*Keywords:* Asymmetric Creep, Refractories, Finite Elements, Modeling

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## 1 List of Symbols

$\underline{\underline{\sigma}}$	Second rank stress tensor
$\sigma_{eq}$	Von Mises equivalent stress
$\sigma_x$	Stress in direction $x$
$\sigma_y$	Stress in direction $y$
$f_y$	Yield stress
$\underline{\underline{s}}$	Second rank deviatoric stress tensor
$\sigma_t$	Tensile strength
$\sigma_c$	Compression strength
$\varepsilon_x$	Total strain in direction $x$
$\varepsilon_y$	Total strain in direction $y$
$p$	Equiv. creep (viscoplastic) strain
$\underline{\underline{\dot{\varepsilon}}}^{cr}$	Second rank creep (viscoplastic) strain rate tensor
$E$	Young's modulus
$\nu$	Poisson's ratio
$A, n, m$	Temperature dependent creep parameters
$w$	Weights
$e$	Sample's thickness
$\phi_s$	Sample's diameter
$\phi_j$	Jaws' diameter
$\underline{\underline{I}}$	Identity matrix
$Tr()$	Trace
$\langle \ \rangle$	Macauley brackets
$x^\pm$	Positive and negative parts of variable $x$
$F$	Force
$t$	Time

## 3 1. Introduction

4 Refractories are materials designed to work at highly aggressive environ-  
5 ments, and need to resist to thermo-mechanical stresses and strains, corrosion  
6 and erosion at temperatures that can reach over 2000 °C (Banerjee, 2004). In  
7 the production of steel, cement, glass, copper, between other materials processed  
8 at high temperatures, the goal of refractory linings is to protect the vessels used  
9 in the production process, normally made of steel, from overheating and conse-  
10 quent mechanical failure, as well as to control the heat losses from the process  
11 (Schacht, 2004).

12 The transformation of raw materials into finished steel products involve high  
13 temperature processes, and different vessels lined with refractories are used to  
14 contain the molten metals, such as blast furnaces, basic oxygen furnaces (BOFs),  
15 electric arc furnaces (EAFs), ladles and tundishes. Among those vessels, the  
16 steel ladle has a considerable importance, since it is used in the secondary met-  
17 allurgy, where a considerable amount of time and financial resources have al-  
18 ready been previously employed on the production process, and is responsible  
19 for a non-negligible consumption of refractories (Dutta & Chokshi, 2020). The  
20 integrity of the ladle is of central importance to the safety of the production  
21 site, since it transits between other equipments and workers, carrying molten  
22 steel, and a failure can lead to serious consequences.

23 Figure 1a shows a cutaway view of a steel ladle, and Figure 1b shows a  
24 schematic representation of the top view of one ring of a ladle lining. The choice  
25 of the refractories to be applied in each layer of the lining greatly influences the  
26 temperature distribution in the ladle, therefore playing also a crucial role on  
27 its thermo-mechanical behavior (Volkova & Janke, 2005) and on the energy  
28 consumption.

29 An important phenomena to be considered in the design of the steel ladle is  
30 the creep and stress relaxation of the refractory lining. The absence of a non-  
31 linear viscoplastic behavior can lead to stress values that can't be supported by  
32 the vessel, causing a mechanical failure. Nevertheless, if the material presents

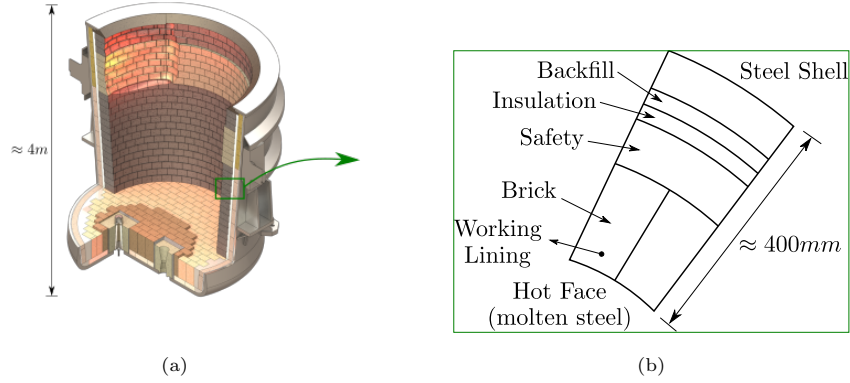


Figure 1: Example of a steel ladle. (a) Cutaway view showing the refractory linings. (b) Schematic representation of a line ring, top view.

excessive creep, it will have a considerable reduction on its size caused by the compression stresses at the hot face. During the cooling of the ladle, joints may open, making the lining loose and allowing for steel infiltration in posterior production cycles. Therefore, the design of the refractory lining and of the steel shell can benefit from a numerical model that can predict more accurately the stress and strain variations over time, and the effect of thermal cycles can be taken into account to maximize the vessel's life.

Due to the complexity of their mechanical behavior, the modeling and simulation of refractory materials has often been restricted to the application of simple constitutive models, in some cases even neglecting the well known non-linearity of the material (Jin et al., 2011). In other cases, creep models have been applied to simulate the thermo-mechanics of refractories, but considering a symmetric behavior and using the compressive mechanical properties (Jin et al., 2020). There were also attempts to use a Drucker-Prager-based creep model to simulate the effect of the material's asymmetry of RH-degassers lining behavior, but the authors concluded that this model might not be ideal and that more complex representation of the actual behavior should probably be used (Jin et al., 2015).

In this work, an asymmetric creep model is proposed, which is able to repre-

sent the different behavior that refractory materials present under tension and compression. The main goal is to obtain a relatively simple constitutive model, that can have the material parameters identified using the least possible number of mechanical tests, due to the need to characterize the material at several temperatures, which requires a high time and energy consumption. With the proposed model, the lifetime of refractory materials can be predicted more accurately than the using current engineering practice (linear elastic or symmetric creep models), since it accounts for the significant increase in the creep strain rate when tensile loads are present in the lining. Similarly, the model allows for a better comparison between the performance of products with different shapes and compositions, reducing field trials and improving the availability of the equipments.

Section 2 provides a description of the current creep models available in the literature, explaining what are the missing features when the simulation of refractory materials need to be done. Section 3 describes the proposed model, and comments on possible numerical difficulties that can arise when implementing it, as well as numerical strategies that can be used to mitigate them. Section 4 presents numerical simulations used to theoretically validate the proposed model. Section 5 describes the high temperature experimental procedure used to characterize the material, with a detailed explanation of the reason why the Brazilian test method was used. Finally, Section 6 presents the identification of the material parameters of an alumina-spinel brick used in steel ladles.

## 2. Symmetric and asymmetric creep models

The creep behavior of ceramic materials can be split in three stages. The first stage, called primary creep, presents a time-dependent strain rate which decreases with time. In the secondary creep stage, the strain rate is considered to be constant, and an approximate equilibrium between hardening and softening processes can be assumed (Naumenko & Altenbach, 2007). Finally, in the third creep stage, the strain rate increases with time until the failure of the material

81 (Jin et al., 2014).

## 82 2.1. Symmetric creep models

83 One of the most traditional creep models available in the literature is based  
 84 on the one-dimension Norton-Bailey’s law, known on its multi-dimensional ver-  
 85 sion as Odqvist’s law (Lemaître & Chaboche, 1990), which relates the creep  
 86 strain rate  $\dot{\underline{\underline{\epsilon}}}^{cr}$  to the stresses using the following equation:

$$\dot{\underline{\underline{\epsilon}}}^{cr} = \frac{3}{2} \underline{\underline{s}} A \sigma_{eq}^{n-1} p^m \quad (1)$$

87 where  $\underline{\underline{s}}$  is the deviatoric component of the stress tensor,  $\sigma_{eq}$  is the von Mises  
 88 equivalent stress,  $p$  is the equivalent viscoplastic strain (sometimes referred to as  
 89 equivalent creep strain) and  $A$ ,  $n$ , and  $m$  are temperature dependent material  
 90 parameters. For the case of secondary creep,  $m = 0$ . The Norton-Bailey’s  
 91 law have been extensively used to characterize refractory materials, due to its  
 92 simplicity and good fitness to experimental results (Jin et al., 2014; Samadi  
 93 et al., 2020; Schachner et al., 2019; Sidi Mammar et al., 2016; Teixeira et al.,  
 94 2020).

95 If an elastic region needs to be considered in the model, a simple modification  
 96 in Equation 1 can be done to take it into consideration (Lemaître & Chaboche,  
 97 1990):

$$\dot{\underline{\underline{\epsilon}}}^{cr} = \frac{3}{2} \frac{\underline{\underline{s}}}{\sigma_{eq}} A \langle \sigma_{eq} - f_y \rangle^n p^m \quad (2)$$

98 where  $f_y$  is the yield stress and  $\langle \ \rangle$  are the Macaulay brackets.

99 It should be noted that different uniaxial laws other than Norton-Bailey’s law  
 100 can be applied to relate the stresses to the creep strains (Lemaître & Chaboche,  
 101 1990). The choice of which law should be used depends on how accurately it  
 102 fits the mechanical tests.

## 103 2.2. Asymmetric creep models

104 As can be noticed in Equations 1 and 2, the Norton-Bailey creep model  
 105 is not able to represent materials with different behaviors under tension and

106 compression (asymmetric creep).

107 The asymmetry in the material's response was approached by Altenbach  
 108 (2001) using a creep strain rate formulation that depends on three linear inde-  
 109 pendent invariants of the stress tensor. Alternatively, Mahnken (2003) proposed  
 110 an asymmetric creep model that uses a scalar variable expressed in terms of the  
 111 ratio of the second and third invariants of the deviatoric stress tensor, called  
 112 stress mode angle in the octahedral plane in the deviatoric stress space.

113 Although these models are mathematically consistent and were successfully  
 114 applied, they present the inconvenient to require a considerable number of ex-  
 115 periments to determine the creep parameters, since tension and compression  
 116 behaviors are not decoupled. For the case of refractories, that need to be char-  
 117 acterized at several temperatures, this represents a limitation.

118 More recently, Samadi et al. (2021) coupled the asymmetric primary creep  
 119 behavior of refractories with a damage model. In this model, the equation for  
 120 the creep strain rate is selected according to the sign of the principal stresses,  
 121 without any particular weighting strategy, and the equations are solved using  
 122 an explicit integration scheme.

123 To account for the asymmetric creep of refractories, Blond et al. (2005)  
 124 extended the Norton-Bailey's model, using a split of the principal stress vec-  
 125 tors into a positive and a negative parts, to propose a secondary creep model,  
 126 resulting in:

$$\underline{\underline{\sigma}} = \langle \underline{\underline{\sigma}} \rangle - \langle -\underline{\underline{\sigma}} \rangle \quad (3)$$

127 This split results in the definition of the model in terms of independent ten-  
 128 sion and compression parameters. In this sense, the two parts of the deviatoric  
 129 stress tensor are given by:

$$\underline{\underline{s}}^{\pm} = \langle \pm \underline{\underline{\sigma}} \rangle - \frac{1}{3} Tr(\langle \pm \underline{\underline{\sigma}} \rangle) I \quad (4)$$

130 where the indexes  $\pm$  indicate the positive and negative parts of the variables,

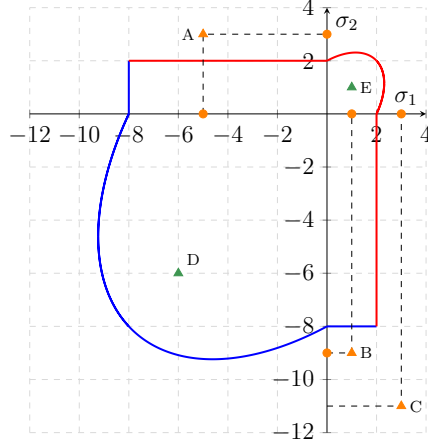


Figure 2: Yield surface of Blond's model.

respectively. The equivalent von Mises stresses are, then:

$$\sigma_{eq}^{\pm} = \sqrt{\frac{3}{2} \underline{s}^{\pm} : \underline{s}^{\pm}} \quad (5)$$

resulting in a viscoplastic strain rate of the form:

$$\dot{\underline{\epsilon}}_{cr} = \frac{3}{2} \frac{\underline{s}^{+}}{\sigma_{eq}^{+}} A^{+} \langle \sigma_{eq}^{+} - f_y^{+} \rangle^{n^{+}} - \frac{3}{2} \frac{\underline{s}^{-}}{\sigma_{eq}^{-}} A^{-} \langle \sigma_{eq}^{-} - f_y^{-} \rangle^{n^{-}} \quad (6)$$

where  $A^{\pm}$  and  $n^{\pm}$  are material's constants in tension (+) and compression (-).

Figure 2 shows a representation of Blond's model in the 2D principal stresses domain. In this example, the tensile yield stress is  $f_y^{+} = 2$  MPa and the compressive yield stress is  $f_y^{-} = -8$  MPa. In the figure, the stress states  $D$  and  $E$  marked as green triangles are inside the elastic region, therefore no creep is observed.

Point  $A$  shows a stress state in the second quadrant, with a compressive component in direction 1 and a tensile component in direction 2. Therefore, the stress split described by Equation 3 results in other two stress states, shown as circular markers. As can be seen, the point  $(-5, 0)$  is in the elastic region, so no creep is expected for the compressive component. On the other hand, the point  $(0, 3)$  lays outside the elastic region, so the first term of the right hand side of



Equation 6 is activated, and creep in tension is observed.

Point B corresponds to the opposite situation, i.e., creep is activated in compression but not in tension. The stress state represented by Point C results in creep in both tension and compression.

From Figure 2, it is also evident that Blond’s yield surface presents singularities at the points where it crosses axis 1 and 2, as well as in the corners present in the second and fourth quadrants. When the structure being modeled has stress states close to these singularities, difficulty in the convergence may arise. Nevertheless, this should not be a problem if the yield stress is equal to zero, i.e., the material starts to creep as soon as a load of any magnitude is imposed, which is often the case for refractories (Jin et al., 2014).

One of the main advantages of this model is that the material’s parameters for tensile and compression creep are completely independent, therefore they can be identified separately. This gives flexibility to fit the creep flow calculations to a large range of strain rates observed in experiments.

Nevertheless, the validity of the stress split hypothesis needs to be verified, and this model presents the characteristic of making a sum between two terms (positive and negative strain rates) that depend on the equivalent stress. The consequence of this fact is that, if this model is used with identical properties in tension and in compression, the resulting viscoplastic strain rate is not the same as the one obtained with a symmetric model. Although this is not necessarily a limitation, in some cases it can be desirable to have a model that is an interpolation of the tension and the compression material behaviors, without this intrinsic orthotropy.

As remarked by Esposito and Bonora (2011), in many applications the primary creep of materials can’t be neglected, since a considerable part of the allowable design strain occurs in this stage. This is the case of the alumina-spinel material studied in this work, as it is evident by the creep curves previously published by Samadi et al. (2020) and Teixeira et al. (2020).

### 174 **3. Proposition of an asymmetric creep model considering primary** 175 **and secondary creep stages**

#### 176 *3.1. Model description*

177 In this work, to represent the primary and secondary creep behaviors of  
 178 refractory materials, Equation 1 was adapted following the same principle of  
 179 the split of the stress tensor in a positive and a negative part, as used by Blond  
 180 et al. (2005).

181 The proposed model also differentiates from Blond's model in the way to  
 182 consider the different contributions of the compression and tensile characteristics  
 183 of the material. After the decomposition of the stress tensor, the deviatoric and  
 184 equivalent stresses are calculated for each part (positive and negative) using  
 185 Equations 4 and 5, respectively, such as in the model proposed by Blond et al.  
 186 (2005).

187 Nevertheless, instead of using  $\underline{s}^\pm$  and  $\sigma_{eq}^\pm$  to directly calculate the positive  
 188 and negative viscoplastic strain rates (Equation 6), these values are used to  
 189 calculate relative weights that each part of the stress tensor have on the total  
 190 equivalent stress, using the relation:

$$w^\pm = \frac{\sigma_{eq}^\pm}{\sigma_{eq}} \quad (7)$$

191 Each portion of the viscoplastic strain rate is calculated as a function of  
 192 the total deviatoric and equivalent stresses (using the full stress tensor, before  
 193 the decomposition into positive and negative parts) and the respective material  
 194 properties:

$$\underline{\dot{\epsilon}}^{cr^\pm} = f(\underline{s}, \sigma_{eq}, A^\pm, n^\pm, m^\pm) \quad (8)$$

Later, each part of the viscoplastic strain rate is weighted by the values calculated using Equation 7. Therefore, the viscoplastic strain rate of the proposed asymmetric creep model is given by:

$$\underline{\dot{\epsilon}}^{cr} = w^+ \cdot \frac{3}{2} \frac{\underline{s}}{\sigma_{eq}} A^+ \langle \sigma_{eq} - f_y^+ \rangle^{n^+} p^{m^+} - w^- \cdot \frac{3}{2} \frac{\underline{s}}{\sigma_{eq}} A^- \langle \sigma_{eq} - f_y^- \rangle^{n^-} p^{m^-} \quad (9)$$

195 This model is also similar to the one developed by Mahnken (2003), in the  
 196 sense that it proposes a weighted calculation of the creep strain rate. Never-  
 197 theless, the model proposed here can be considered to be more straightforward,  
 198 and it also requires less parameters to characterize the creep behavior of a given  
 199 material, [which](#) is an important advantage. This model was implemented in an  
 200 Abaqus UMAT subroutine, and examples of calculations are shown in the next  
 201 sections. The integration algorithm according to Benallal et al. (1988) was used  
 202 in this subroutine.

203 It should be [noted](#) that, when the material presents primary creep under  
 204 compression and secondary creep under tension, such as in the case of the  
 205 alumina-spinel brick,  $m^- = 0$  in Equation 9. [To summarize, Figure 3 presents](#)  
 206 [flow charts corresponding to the algorithm proposed by Blond et al. \(2005\) \(full](#)  
 207 [lines\) and the algorithm proposed in this work \(dashed lines\).](#)

### 208 3.2. Numerical difficulties associated with primary creep

209 At the beginning of a structural simulation, when the load was not yet  
 210 applied, the strain in the simulated body is assumed to be zero, unless otherwise  
 211 stated. In this situation, it is reasonable to assume that the equivalent creep  
 212 strain is  $p = 0$ . Examining Equation 9 and taking into account that the variable  
 213  $m$  can only take negative values, it is easy to deduce that:

$$\lim_{p \rightarrow 0} \underline{\underline{\dot{\epsilon}^{cr}}} = 0 \quad (10)$$

214 Therefore, the equivalent viscoplastic strain can't be defined as zero at time  
 215  $t = 0$ , since this would result in the absence of creep strain in the subsequent  
 216 time step, independently of the applied load.

217 The first solution to this problem is to define a low non-zero initial value  
 218 for  $p$ . Nevertheless, it is not always clear which value should be used, since the  
 219 convergence of the simulation is highly dependent on it. For example, consider a  
 220 case where only compression creep is applied and  $f_y = 0$ . Equation 9 becomes:

$$\underline{\underline{\dot{\epsilon}^{cr}}} = \frac{3}{2} \underline{\underline{s}} A^- \sigma_{eq}^{(n^- - 1)} p^{m^-} \quad (11)$$

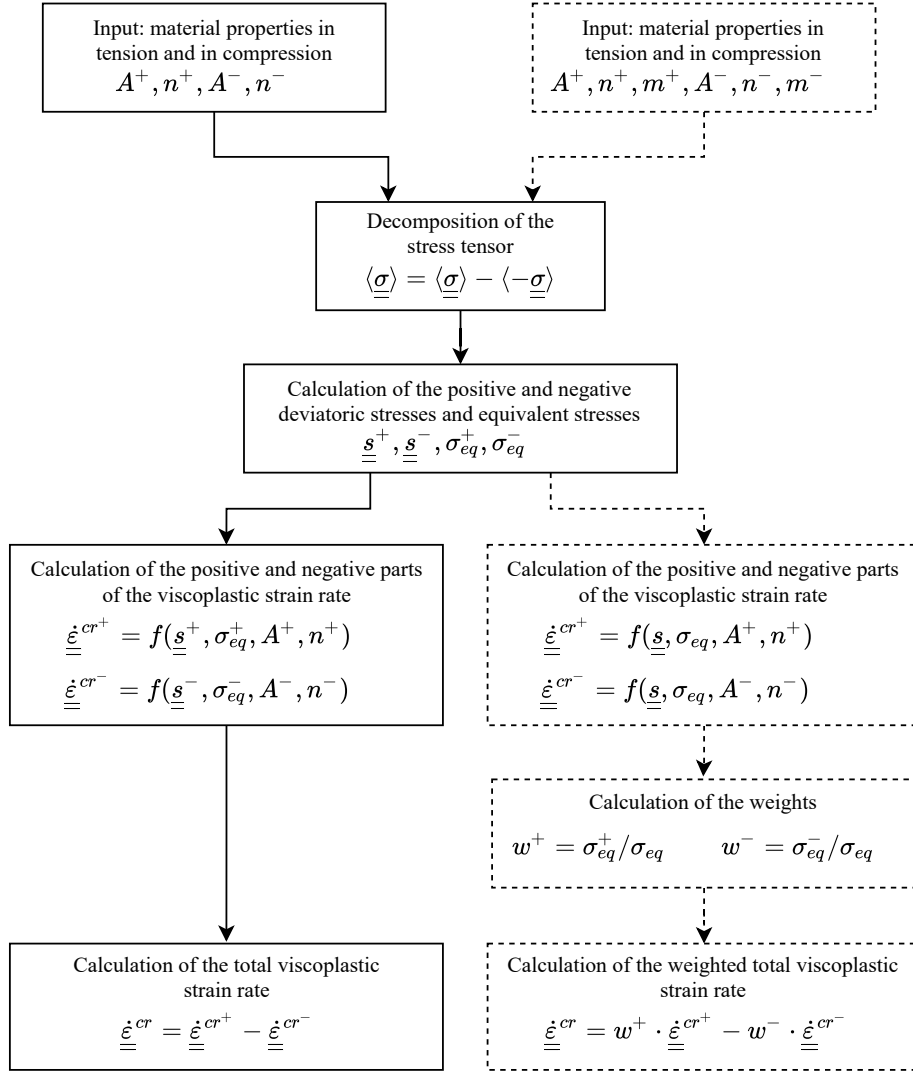


Figure 3: Asymmetric creep model algorithms. Blond et al. (2005) (full lines) and the proposition of this work(dashed lines)

221      Supposing  $\underline{s}$  and  $\sigma_{eq}$  to have an order of magnitude of 10,  $A \approx 1 \times 10^{-10}$ ,  
 222       $m \approx -2$ ,  $n \approx 2$  and an initial approximation of  $p = 1 \times 10^{-15}$ , the resulting creep  
 223      strain rate would be  $\underline{\dot{\epsilon}}^{cr} \approx 1 \times 10^{22}$ . A high value of initial creep strain rate is

224 expected from primary creep, but at this rate of strain, the integration algorithm  
225 would have to use an extremely low time step value in order to converge, which  
226 is not practical.

227 It can be concluded that an initial equivalent viscoplastic strain value should  
228 be used so that it is small enough to do not decrease the accuracy of the solution  
229 and large enough to allow the convergence in a reasonable time frame. This value  
230 depends on the material properties, the applied loads, the boundary conditions  
231 and on the time stepping used during the solution, and can be considerably  
232 difficult to estimate.

233 To improve the convergence of the model, the following actions can be taken:

- 234 1. Given the material properties and boundary conditions of the problem, an  
235 initial value for the equivalent creep strain  $p_{t=0}$  needs to be determined.  
236 The convergence of the initial steps of the model is tested, using a small  
237 time increment. If necessary,  $p_{t=0}$  should be increased.
- 238 2. The time step increment should be small during the first seconds of load-  
239 ing, since the primary creep curve can be steep at the beginning. It can  
240 be interesting to brake the initial steps into smaller ones, to allow a finer  
241 control of the time increment.
- 242 3. Even when the initial moments of primary creep are past, the maximum  
243 increment of time should be limited to reasonable values, since the auto-  
244 matic time stepping control present in some FEA software can attempt  
245 to increase it beyond the convergence limit of the integration algorithm.  
246 Even if the software is able to converge, it can take many iterations to  
247 finish a step, leading to slower computations.

248 Even if those measures are taken, the simulation can still diverge due to  
249 a rapid change in the creep strain rate, specially for the case where tensile  
250 loads are meaningful. To limit this problem, in the UMAT subroutine used to  
251 implement the creep models developed in this work, a variable was implemented  
252 to control the convergence of the integration algorithm. If the algorithm does  
253 not converge, the subroutine automatically returns to the previous converged

iteration, and tries a time step that is half the previous attempt. This action is only effective if the convergence problem is related to the integration algorithm of the constitutive equations, not to the global Newton-Raphson integration scheme.

#### 4. Numerical simulations using the proposed asymmetric creep model

To evaluate the capabilities of the proposed asymmetric model, a set of numerical simulations is presented, in increasing order of complexity. The goal of these simulations is to verify if the model presents the expected behavior when subjected to complex load cases. The material parameters used to perform these tests are shown in Table 1. As it will become evident in the next sections, these material parameters are close to the one corresponding to the alumina-spinel material. It was considered that the tensile behavior of the material could be approximated by a secondary creep model, as discussed by Teixeira et al. (2020).

Table 1: Material parameters used in the numerical simulation tests

Parameter	Compression	Tension
$E[\text{MPa}]$	30000	30000
$\nu[-]$	0.2	0.2
$\log_{10} A[\text{MPa}^{-n} s^{-1}]$	-14.16	-5.4
$n[-]$	3.96	1.5
$m[-]$	-2.74	-

In the simulations, four situations regarding the type of model and the material parameters were considered:

1. Abaqus symmetric creep model using the compression properties of the material. This configuration is commonly seen in publications related to the creep of refractories, as previously cited.

- 272 2. Abaqus symmetric creep model using the tension properties of the mate-  
273 rial, used as a reference to compare with the asymmetric model.
- 274 3. UMAT asymmetric creep model, but using the compression properties  
275 of the material for compression and tension, to verify if the asymmetric  
276 model specializes to a symmetric one when necessary.
- 277 4. UMAT asymmetric creep model, using the corresponding properties for  
278 tension and compression.

#### 279 4.1. Normal loads with stress reversal in two dimensions

280 The first model corresponds to a simple two-dimensional element (Figure 4a,  
281 where  $e$  is the thickness) subjected to a load path that varies in directions  $x$  and  
282  $y$ , according to Figure 4b. The maximum tensile stress is  $\sigma_t = 0.2$  MPa, and  
283 the minimum compression stress is  $\sigma_c = -2.0$  MPa. Figures 4c and 4d show  
284 the time periods for which the stresses are kept in the sample.

285 Although this is a simple model, it represents a situation where, during  
286 the loading cycle, there are moments where both principal stresses are positive  
287 (Point B), both are negative (Point D), and when there is a composition of  
288 positive and negative stresses at the same integration point (Points C and E).  
289 Therefore, the simulation is useful to show the difference between a symmetric  
290 and an asymmetric model, as well as the effect of the loading history. For this  
291 model, an extra curve using the tension material parameters for both tension  
292 and compression material laws is also presented.

293 Figure 5 shows the accumulated viscoplastic strains computed in each of the  
294 situation previously described. It is possible to observe that, when the same  
295 properties are used for tension and compression, the UMAT provides the same  
296 results as a symmetric model. More importantly, the asymmetric model presents  
297 an intermediate response between the tension and the compression symmetric  
298 models, as expected.

299 In Figure 5 it is clear that, until approximately 30 min, when only tension  
300 stresses are present in the element (Points A and B in Figure 4), the symmetric  
301 and asymmetric models give the same result. From that point further, when

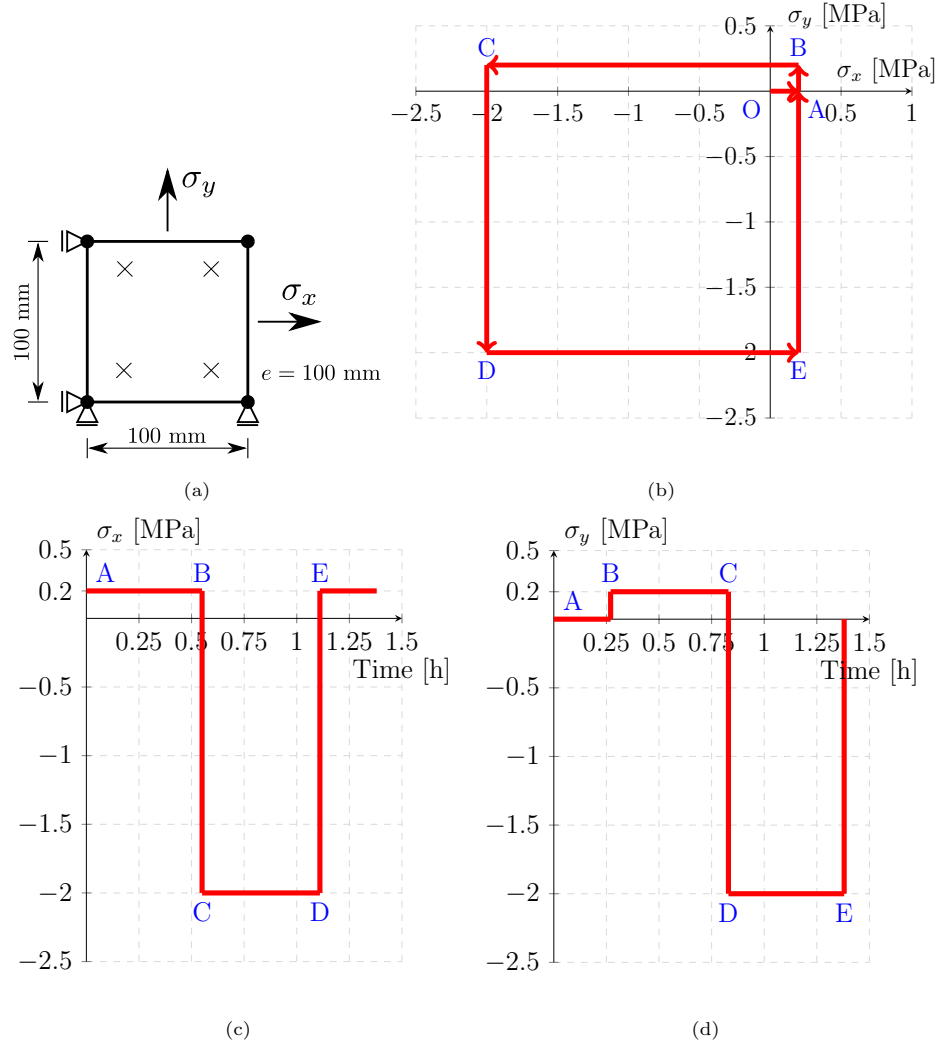


Figure 4: Stress distribution – Normal loads. (a) Simulation model. (b) Stress path. (c)  $\sigma_x$  vs Time. (d)  $\sigma_y$  vs Time.

302 an asymmetry is included in the loading (Point C in Figure 4), the model re-  
 303 sponse changes, becoming an interpolation between the tension and compression  
 304 behaviors.



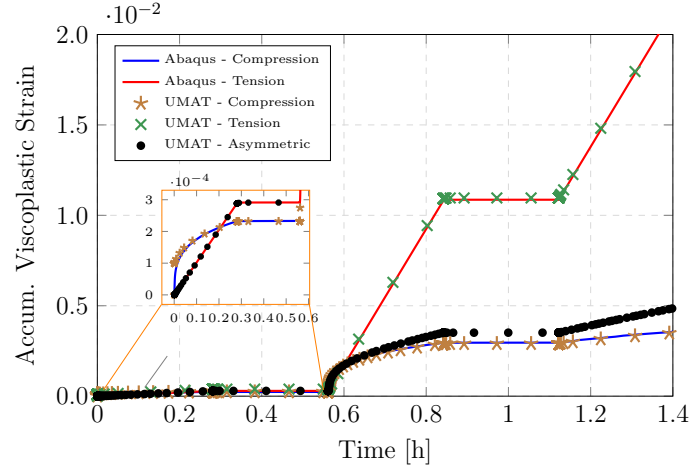


Figure 5: Accumulated viscoplastic strain — Normal loads.

#### 4.2. Brazilian test

Figure 6 shows the geometry, mesh and boundary conditions used to compare the symmetric and asymmetric models applied to a Brazilian test specimen. The sample was discretized using linear square elements with full integration, unless for a transition zone between the refined mesh in the contact region and the rest of the geometry, where linear triangular elements with full integration were used.

The sample was modeled with  $\phi_s = 50$  mm diameter and  $e = 40$  mm thickness. Analytic rigid surfaces representing the upper and lower jaws with diameters  $\phi_j = 65$  mm were used to distribute the load more evenly on the sample and to avoid excessive stress concentrations. The same strategy was used at the bottom part of the model to restrict the vertical displacement of the sample. Due to the geometry and force symmetries, only half of the sample was modeled.

A force of  $-400$  N was applied on the model following a 30 s linear ramp, and it was kept for two hours. This geometry and boundary conditions are the same used for the mechanical tests at  $1300$  °C, presented in Section 5.

A comparison between vertical and horizontal displacements taken at the center of the sample for the four cases is shown in Figure 7. It is possible to

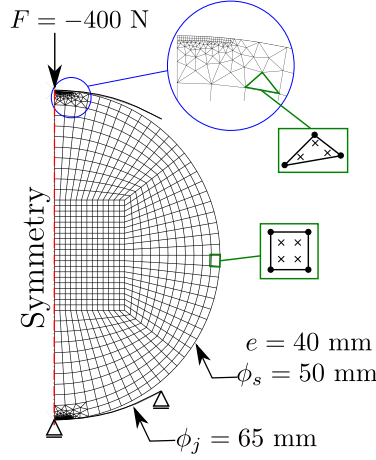


Figure 6: Brazilian test – Geometry, mesh and boundary conditions.

323 observe that, when compression curves are used in the asymmetric model, the  
 324 result is in high agreement with the symmetric model available on the software  
 325 Abaqus.

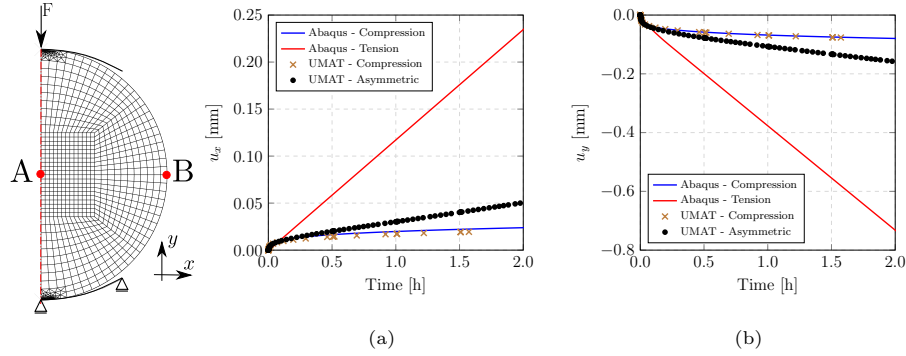


Figure 7: (a) Vertical and (b) horizontal displacements in the Brazilian test sample.

326 Figures 8 and 9 show the maximum principal stress and the minimum prin-  
 327 cipal stress on the sample, respectively, for the cases where only tension or  
 328 compression material parameters were considered (symmetric creep), and the  
 329 asymmetric case. It can be observed that the maximum tensile stress value  
 330 obtained on the sample was 0.35 MPa. Since refractory materials are less re-

331 sistant to tension, this stress limits the amount of force that can be applied  
 332 before the failure of the sample. The maximum compressive stresses observed  
 333 on the sample are in the range of  $-2.5$  MPa to  $-3.8$  MPa, although at the  
 334 center of the sample this value does not go higher than  $-1.8$  MPa. This can be  
 335 considered as a limitation of the Brazilian test when compared to uniaxial tests  
 336 for the identification of material parameters, i.e., the tension and compression  
 337 loads are not decoupled, therefore the material can only be safely character-  
 338 ized for a smaller stress range. Nevertheless, this only represents a problem if,  
 339 during operation, the actual compression stresses withstand by the material are  
 340 considerably higher than the ones during the mechanical tests.

341 It should be noted that, in Figures 8 and 9, the contact zones were excluded  
 342 from the results plots. This was done because, in these regions, the model can  
 343 experiment stress concentrations, that makes the visualization of the plots more  
 344 difficult in the other regions of the model. In a real experiment, these regions  
 345 suffer from a local failure, with the crash of the grains at the contact with  
 346 the jaws. This failure can normally be neglected in the experiment, but it can  
 347 influence the convergence of the numerical model. For instance, the elements  
 348 with higher stresses present, as a consequence, higher creep strain rate, and the  
 349 model takes longer to converge due to the need of low time steps, even if the bulk  
 350 part of the sample is still under low stresses. To use the proposed asymmetric  
 351 model, attention should be paid to such stresses concentration areas, and they  
 352 should be removed from the model, if possible.

353 When compared to a symmetric model using the compression creep prop-  
 354 erties, the asymmetric model has a considerably different behavior over time  
 355 regarding the strains. Figure 10 shows that, over the central line of the sam-  
 356 ple, the total strain  $\varepsilon_x$ , that is mainly positive, is the same for both models at  
 357 the end of loading (30 s). Nevertheless, after one hour, the strain obtained by  
 358 the asymmetric model is approximately the double of the one obtained by the  
 359 symmetric model, and after two hours the ratio between them is around three.  
 360 The same proportional difference is observed on Figure 11 for the total strain  
 361  $\varepsilon_y$ , that is compressive. This fact is due to the averaged sum of the tension

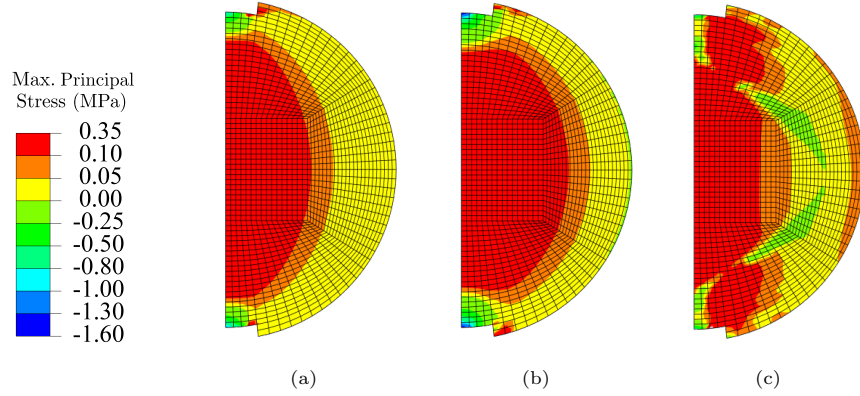


Figure 8: Maximum principal stress distribution on a Brazilian test sample. (a) compression (b) tension and (c) asymmetric material properties.

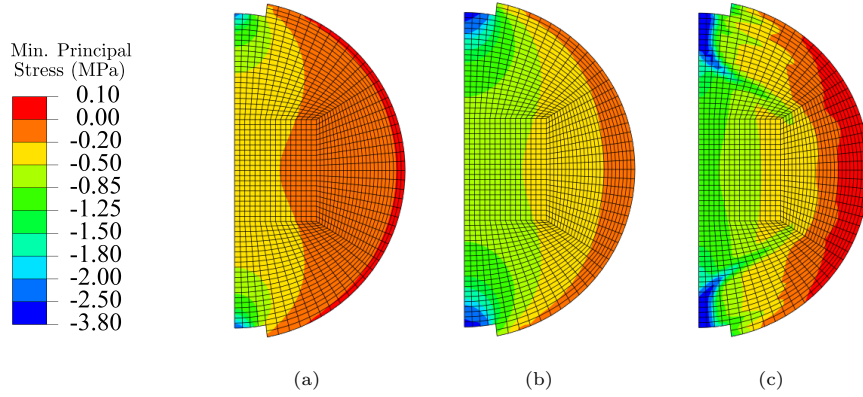


Figure 9: Minimum principal stress distribution on a Brazilian test sample. (a) compression (b) tension and (c) asymmetric material properties.

362 and compression creep strain rates applied by the asymmetric model, where the  
 363 tension part contributes to considerably increase the value of the strains.

364 Finally, Figure 12 shows the effect of the asymmetry in the accumulated vis-  
 365 coplastic strain on a Brazilian test. Once again, the proposed asymmetric model  
 366 shows an intermediary behavior between the symmetric ones, and evidences the  
 367 importance of the consideration of the different material properties in tension  
 368 and in compression.

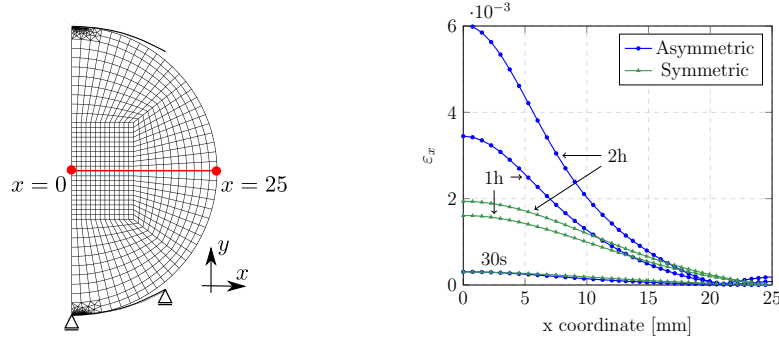


Figure 10: Variation of the total strain in direction  $x$  with time – Symmetric and asymmetric models.

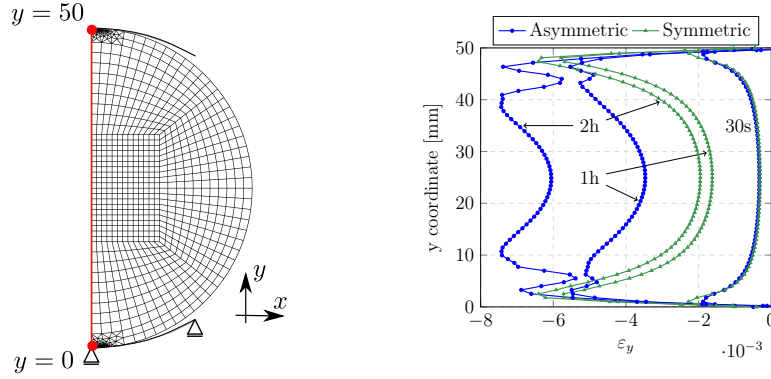


Figure 11: Variation of the total strain in direction  $y$  with time – Symmetric and asymmetric models.

#### 369 4.2.1. Influence of the constitutive model parameters

370 To have a better idea on how each of the constitutive model parameters  
 371 influences the results obtained from a Brazilian test simulation, calculations  
 372 were made varying one parameter at a time around the absolute values of the  
 373 nominal properties presented on Table 1 by  $-10\%$ ,  $-20\%$ ,  $+10\%$  and  $+20\%$ .  
 374 These values are shown in Table 2. This method is not considered an accurate  
 375 sensitivity analysis, and the intention is to qualitatively understand how the  
 376 parameters influence the shape of the time-displacement curves, which is an  
 377 useful information when an inverse identification of the parameters using real

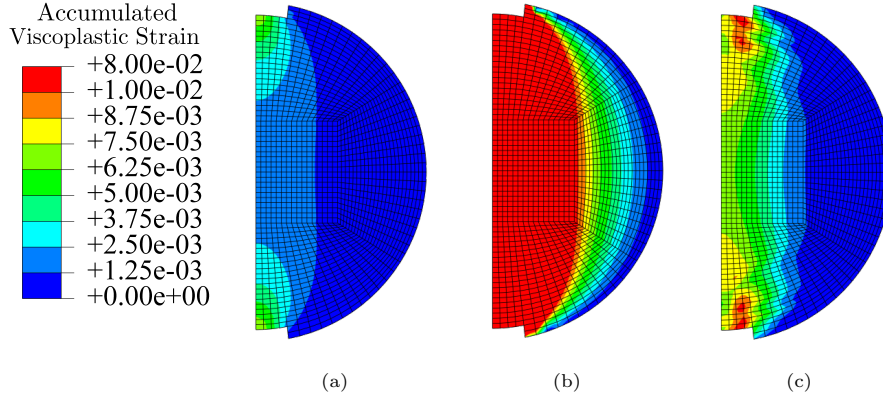


Figure 12: Accumulated viscoplastic strain distribution on a Brazilian test sample. (a) compression (b) tension and (c) asymmetric material properties.

378 experiments needs to be done. Figure 13 shows the variation of the vertical  
 379 displacement  $u_y$  of the top of the sample as a function of time, according to the  
 380 change of the parameters.

Table 2: Variation of the material parameters – Isotropic creep model

Parameter	+10%	+20%	−10%	−20%
$\log_{10} A^- [\text{MPa}^{-n} s^{-1}]$	−15.57	−16.99	−12.74	−11.32
$n^- [-]$	4.35	4.75	3.56	3.16
$m^- [-]$	−3.01	−3.28	−2.46	−2.19
$\log_{10} A^+ [\text{MPa}^{-n} s^{-1}]$	−5.94	−6.48	−4.86	−4.32
$n^+ [-]$	1.65	1.8	1.35	1.2

381 Figures 13a, 13b and 13c show the influence of the compression parameters  
 382  $A^-$ ,  $n^-$  and  $m^-$ , respectively. The curve with the variation of -20% was not  
 383 plotted, since the displacement was excessively high, due to the large influence  
 384 of this parameter in the results. It is possible to observe that parameter  $n^-$ ,  
 385 that is an exponent of the stress, has a negligible influence on the results, due to

the low values of stresses in this application. Parameters  $A^-$  and  $m^-$  influence mostly the beginning of the curve, changing its curvature radius, indicating that the compression stresses are predominant during the first hour, and the tensile stresses take over from this point further. This is an important conclusion, since it helps to decide in which part of the curve the identification procedure will focus, depending on the desired result.

Similarly,  $A^+$  has the higher influence among the tensile parameters, as can be observed in Figures 13d and 13e, although  $n^+$  starts to be influent after one hour of loading and can become more meaningful for longer periods. Contrary to the compressive parameters, the tensile parameters, specially  $A^+$ , change mainly the slope of the curve after the initial curvature has passed. Again, this remarkable separation in time between the influence of the tensile and compressive parameters facilitates the inverse identifications.

## 5. Experimental methodology

In this work, Brazilian tests were used to identify the creep parameters of an alumina-spinel material. Traditionally, Brazilian tests have been applied to study the tensile strength of concretes and geomaterials, as a replacement to direct tensile tests. As pointed by several studies, the results of a Brazilian test to calculate the tensile strength of a given material is only valid if the crack initiation is at the center of the sample, and not at the stress concentrations present at the contact points (Darvell, 1990; Fairhurst, 1964; García et al., 2017). For the applications in the current work, the position of the initiation of the crack is not important, since no fracture is expected during the test.

The Brazilian test was chosen for the identification of the asymmetric creep parameters due to the complexity of the stress field developed in the sample during the application of the load. As shown in Figure 14, tensile stresses are developed at the center of the sample, while compressive stresses start to increase towards its borders and shear stresses are present in the regions in contact with the upper and lower jaws. A more detailed description of the

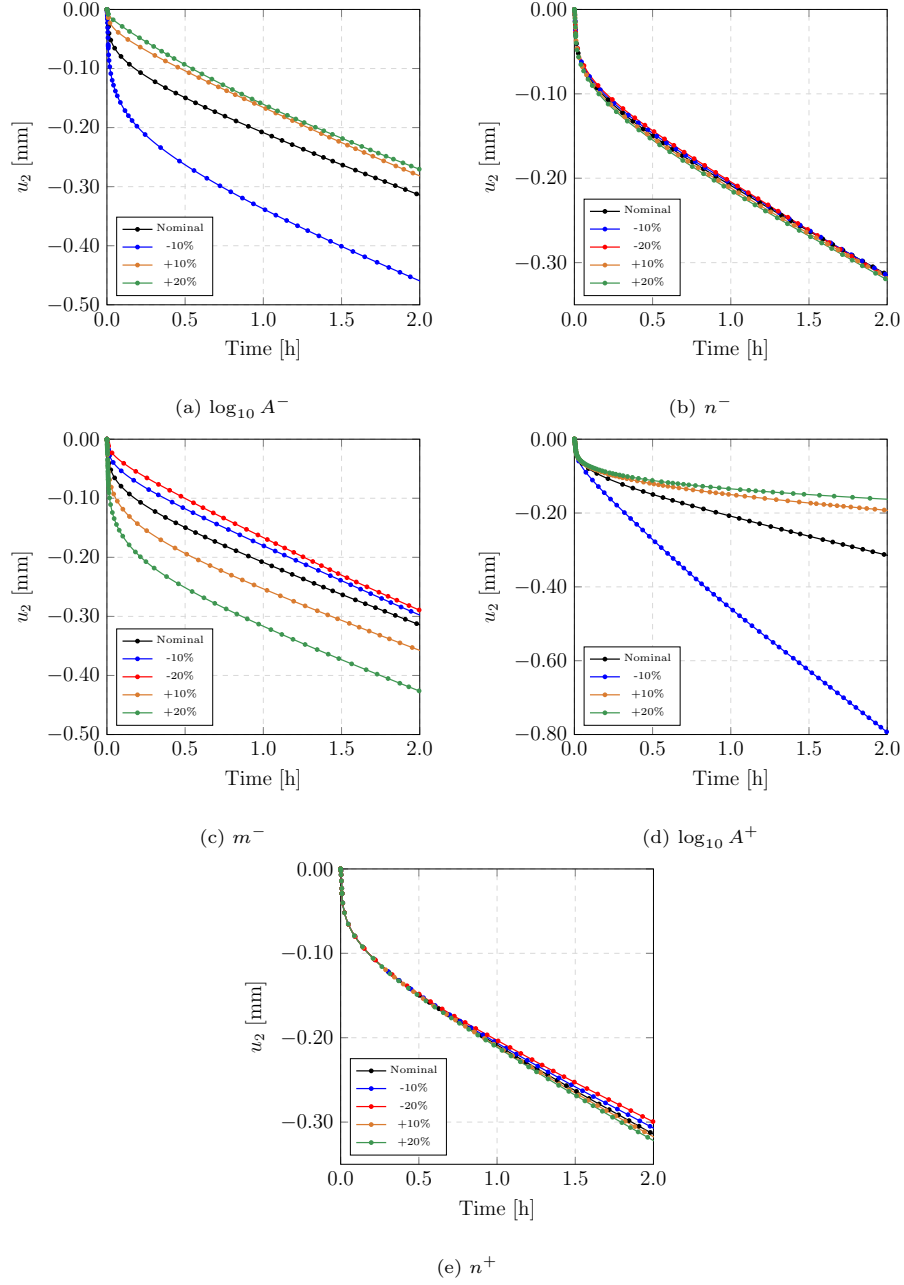


Figure 13: Change on the asymmetric creep model response due to variations of the material parameters – Brazilian test.



415 stress equations related to the Brazilian test can be found in Fahad (1996).  
 416 This stress distribution is adapted to the current case, since it allows [the study](#)  
 417 [of](#) the effect of the material properties under tension and compression using a  
 418 single test, which considerably reduces the number of experiments that need to  
 419 be performed.

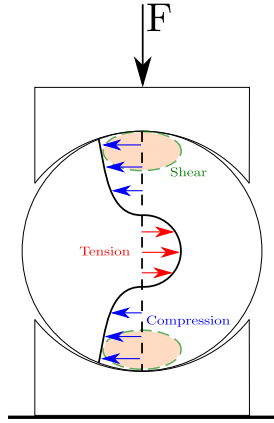


Figure 14: Stress distribution in a Brazilian test sample

420 The application of Brazilian tests to refractory materials is still limited, spe-  
 421 cially regarding the characterization of creep behavior. Belrhiti et al. (2017) used  
 422 this test in association to a DIC technique to characterize the mechanical be-  
 423 havior of magnesia hercynite refractories used in cement rotary kilns, estimating  
 424 the elastic modulus, Poisson's ratio and fracture energy at room temperature.  
 425 Gazeau et al. (2015) used the Brazilian test associated with the integrated DIC  
 426 technique to identify the Young's modulus and the tensile strength of plane  
 427 membranes at temperatures up to 900 °C. These two applications demonstrate  
 428 the potential of this method to provide various information using a single ex-  
 429 perimental setup, what can result in the reduction of the required number of  
 430 tests to characterize a material. This is specially important considering the cost  
 431 and time demand of high temperature tests.

### 432 5.1. Experimental setup

433 In this work, the Brazilian test was used in association with a DIC technique.  
 434 This technique is convenient for the current application, since the traditional  
 435 instrumentation of testing setups at high temperatures present significant diffi-  
 436 culties.

437 Previous works have given detailed information about DIC applications at  
 438 high temperatures (Leplay et al., 2015; Leplay et al., 2010, 2012; Novak & Zok,  
 439 2011). When pictures at high temperatures need to be taken, the excessive  
 440 black body radiation coming from the furnace needs to be filtered, otherwise  
 441 the image is oversaturated and presents no contrast. This is frequently achieved  
 442 by using blue band pass filters, and sometimes neutral density filters depending  
 443 on the exposure time of the pictures, that only allows that the blue part of the  
 444 light spectrum arrives at the camera sensors. To increase the quality of the  
 445 pictures, it is also common to enlighten the sample using a blue light source.

446 Figure 15 shows the experimental setup used to perform the Brazilian tests  
 447 and to take the pictures at high temperatures. This setup is composed of the  
 448 following parts:

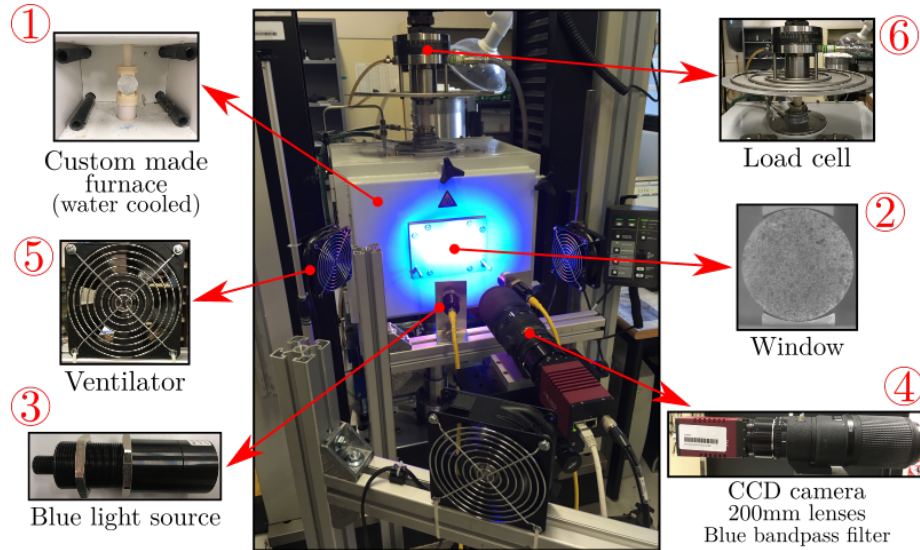


Figure 15: Experimental setup for the Brazilian tests at high temperature

- 449 1. A water cooled custom made furnace with the capacity to heat up to  
450 1300 °C.
- 451 2. The furnace's door is equipped with a window made of a vitro ceramic ma-  
452 terial, so that the sample can be photographed. This material was chosen  
453 because of its adequate resistance to temperatures up to 1300 °C, its low  
454 thermal expansion and its inexpensive price. The material is transparent  
455 enough to allow that satisfactory pictures are taken.
- 456 3. To increase the amount of blue light available when taking the pictures,  
457 two blue light sources are used to enlighten the sample. In combination  
458 with the blue filters, this increases significantly the contrast of the pictures,  
459 to a point where the DIC analysis becomes possible. These lights are  
460 mounted in a dedicated support that was designed to offers six degrees of  
461 freedom, so they can be easily positioned to avoid glare in the pictures.
- 462 4. A high resolution 12 bit CCD camera, AF Micro-Nikkor 200 mm lenses  
463 and a blue band pass filter, to decrease the amount of light being captured  
464 by the camera, since it blocks all parts of the optical spectrum that are  
465 not blue, avoiding the saturation of the sensor.
- 466 5. The camera and the lenses are cooled using two ventilators, since they  
467 stand close to the furnace and can overheat. A third ventilator is used to  
468 blow away the hot air between the window and the lenses, minimizing the  
469 generation of heat hazes.
- 470 6. A load cell with maximum capacity of 30 kN.

471 The samples used in the Brazilian tests were cut from parallel refractory  
472 bricks. The first step was to cut slices 40 mm thick, to guarantee the parallelism  
473 of the sample surface exposed to the camera. Second, 50 mm cylinders were  
474 drilled from these slices. This geometry was chosen to comply with a rule of  
475 thumb on the refractories field that requires the minimum dimension of the  
476 sample to be at least 10 times the size of the largest grain in the material, that,  
477 in the case of the alumina-spinel brick, is 3 mm.

478 To improve the contrast of the samples surfaces, a *SiC* powder speckle pat-

tern was used, with grain sizes varying from 50  $\mu\text{m}$  to 100  $\mu\text{m}$ . The surfaces of the sample were covered with a bonding agent, and the powder was deposited using a sieve, similar to what was done by Archer et al. (2020), what allowed a reasonable control over the particles dispersion. Using this method, the speckle does not remain strongly attached to the sample, and, even if it does not detach under simple gravitational action, direct contact with it should be avoided. Before the test, the sample was let at rest for 12 hours, in order to dry the excess of the bonding agent. Figure 17a shows an example of a sample coated with a speckle pattern, already positioned at the testing machine.

To guarantee the good quality of the pictures, a rigorous procedure was followed at each test. This procedure was composed of:

1. The sample and the jaws were positioned at the testing machine at room temperature, and an initial pre-load of approximately 50 N was applied.
2. The furnace was closed, and the heating of the sample started. Figure 16 shows the heating curves used for the tests. Between room temperature and 600 °C, a heating rate of 10 °C/min was used. From 600 °C to 1300 °C, this rate was reduced to 5 °C, to avoid thermal damage on the sample and on the equipment. Before the beginning of the test, the furnace was let in a dwell for 1 h, to stabilize the temperature of the testing setup. The blue curve on Figure 16, obtained from one of the tests made at 1300 °C, shows that the displacement of the machine piston was constant before the end of the dwell, indicating that there was no extra thermal expansion of the equipment at the beginning of the test.
3. After the temperature homogenization time was passed, the camera lenses and the blue band pass filter were cleaned to remove any dust particles that could cause artifacts in the pictures and later mounted on the camera. To increase the images contrast, the blue lights were positioned to generate the maximum illumination of the sample as possible, avoiding glare, and the focus was manually adjusted in order to increase the sharpness. The exposure time was adjusted according to the lightning conditions to avoid

- 509 too bright or too dark images.
- 510 4. The sample was loaded at a rate of 0.5 mm/min until the creep load was
- 511 achieved, and the force was held constant until the end of the test.

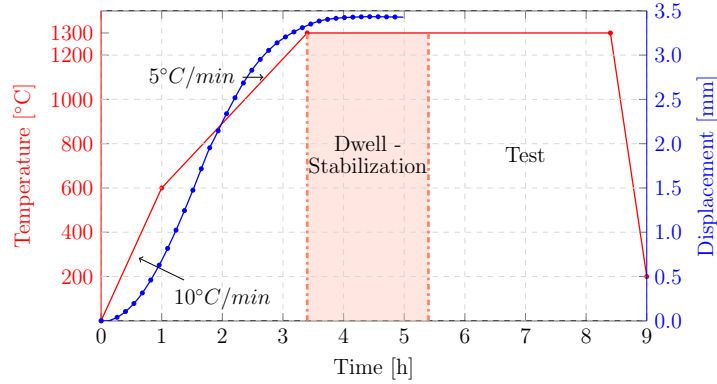


Figure 16: Heating curve and evolution of machine displacements prior to the test.

512 Figure 17 shows the effect of the lightning and of the blue band pass filter

513 in the quality of the images. At room temperature (Figure 17a), the histogram

514 of the image, shown in Figure 18, is approximately normally distributed around

515 gray values from 75 to 200, and the image presents a high contrast. At 1200 °C,

516 without the use of the blue lights and of the blue band pass filter (Figure 17b),

517 the distribution of gray levels is restricted to the range between 105 and 115,

518 and the images loses most of its contrast. The use only of the blue band pass

519 filter, without the blue lights (Figure 17c), slightly decreases the sharpness of

520 the histogram, but not enough to guarantee enough contrast. Finally, using the

521 blue lights and filter (Figure 17d), the gray level distribution becomes closer to

522 that of the image at room temperature, and most of the contrast is recovered.

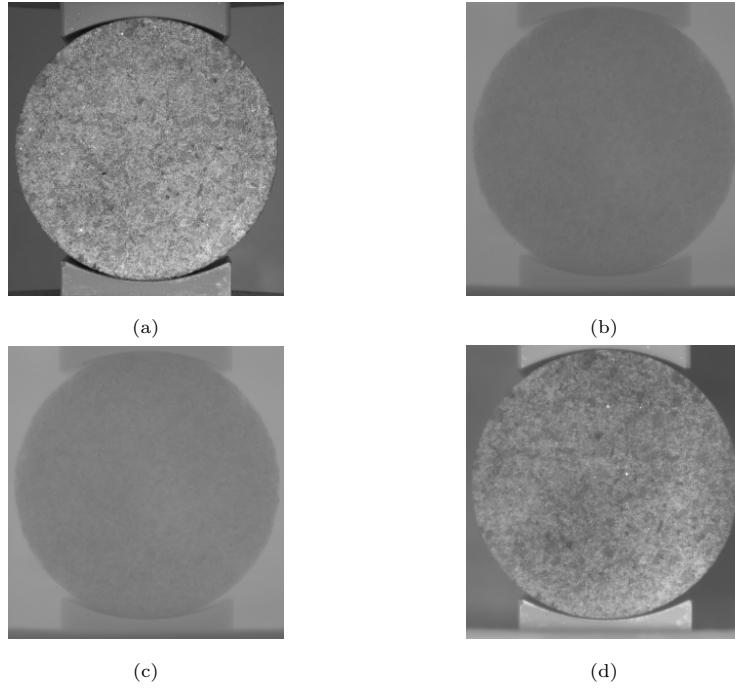


Figure 17: Influence of the blue light and the blue band pass filter in the quality of the image. (a) Image at room temperature. (b) No blue light and no filter. (c) No filter. (d) Use of blue light and blue band pass filter.

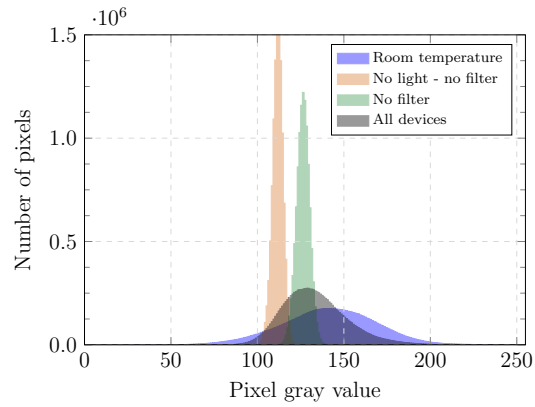


Figure 18: Histograms of the images at room and high temperature.

late the full field displacements of the samples. Ncorr uses the local subset-based reliability-guided DIC method according to Pan (2009).

## 6. Identification of the asymmetric creep properties for the alumina-spinel material at 1300 °C

Figure 19 shows the relative vertical displacement of the upper point of the Brazilian test samples at 1300 °C, calculated using DIC, that corresponds to the difference between the vertical displacements at the upper part and the vertical displacements at the lower part, to account for rigid body movements of the experimental setup. It is possible to observe that samples 1, 3, 5 and 6 are in good agreement, while samples 2 and 4 are lower and upper outliers, respectively.

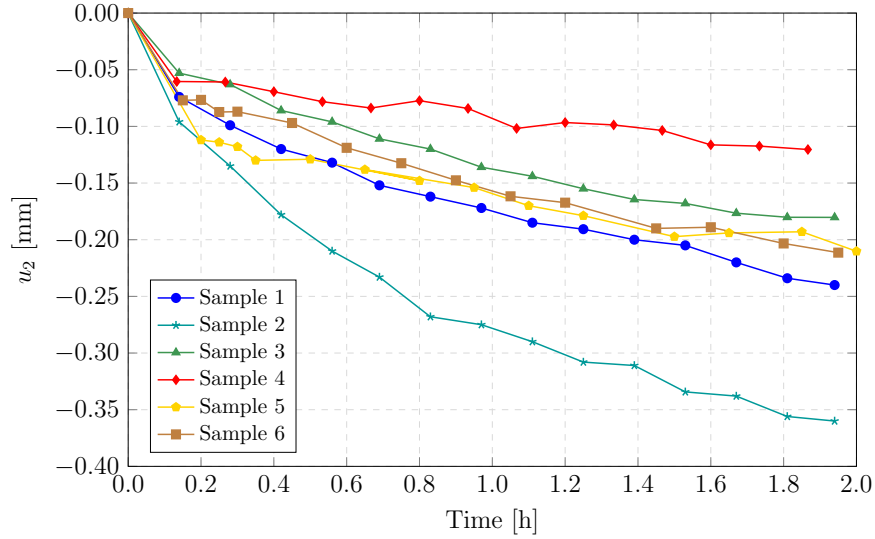


Figure 19: Brazilian tests time vs displacement curves obtained by DIC.

Figure 20 shows an example of envelope for the material parameters at 1300 °C, obtained through a series of numerical simulations using the asymmetric creep model presented in Section 3. It can be expected that the material parameters will not highly deviate from the ones presented in the figure, even

539 if this analysis only considers a single displacement value, and not the full field.

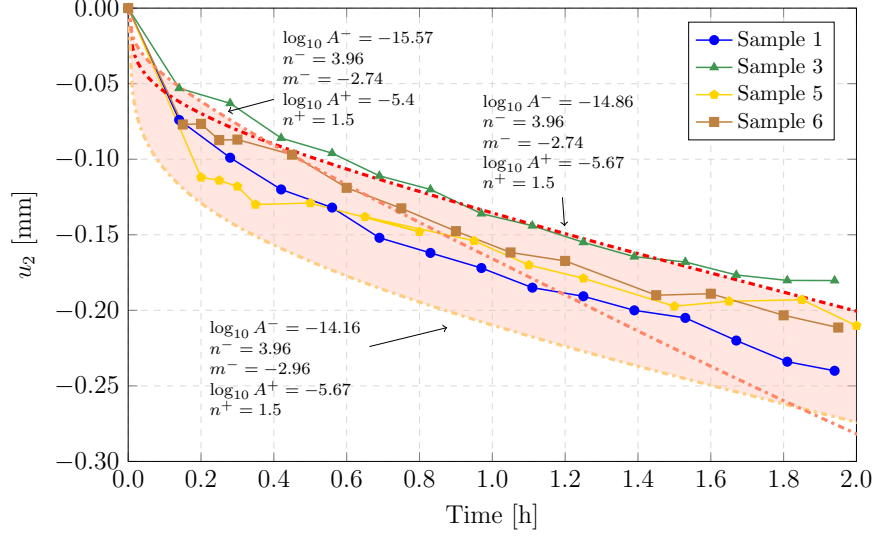


Figure 20: Brazilian tests time vs displacement curves and parameters envelope.

540 Finally, taking into account the influence of the material parameters in each  
541 portion of the time vs displacement curve for Brazilian tests presented in Sec-  
542 tion 4.2.1, the parameters for the alumina-spinel material were identified in  
543 order to better approximate the values of the DIC calculations, and the result-  
544 ing curve is presented in Figure 21. The identified parameters are shown in  
545 Table 3.

Table 3: Identified material parameters

Parameter	Compression	Tension
$\log_{10} A [\text{MPa}^{-n} \text{s}^{-1}]$	-14.86	-5.55
$n [-]$	3.96	1.5
$m [-]$	-2.74	—

546 From Figure 21 it is possible to observe that the identified curve fits the



547 experimental values with good accuracy. The full DIC displacement field of  
 548 sample 6 was compared with the results of numerical simulations performed  
 549 using the identified material parameters, to verify the robustness of the identi-  
 550 fication.

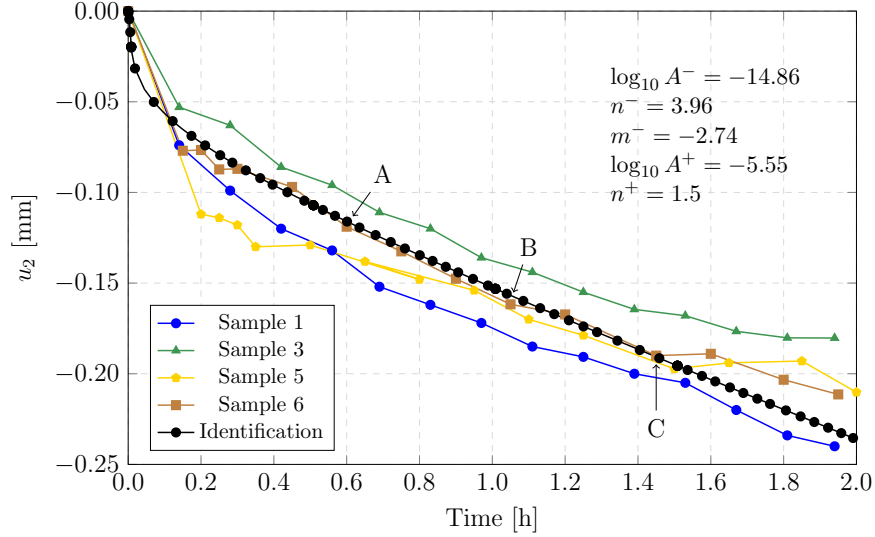


Figure 21: Brazilian tests time vs displacement curves and inverse identification.

551 The vertical displacements fields of Points A, B and C represented in Fig-  
 552 ure 21, corresponding to  $t = 0.6$  h,  $t = 1.05$  h and  $t = 1.45$  h, respectively, are  
 553 shown in Figure 22. It is possible to see that there is a rigid body rotation of  
 554 the sample, since the displacements map does not correspond to the traditional  
 555 displacements field of Brazilian tests. To consider this effect, an extra identifica-  
 556 tion calculation was made to determine what is the magnitude of the load that  
 557 caused this deviation. An horizontal load of  $-7$  N was identified and applied  
 558 on the upper jaws in the simulation model. This corresponds to an error of  $0.5^\circ$   
 559 in the application of the load, which shows that the experimental procedure is  
 560 sensitive to small deviations from the ideal boundary conditions.

561 Figure 23 shows the results of the numerical simulations using the previ-  
 562 ously identified material parameters and the horizontal load. It is possible to  
 563 observe that the displacement maps of Figures 22 and 23 have a good equiv-

564 alence, despite the experimental errors and the simplicity of the identification  
 565 procedure.

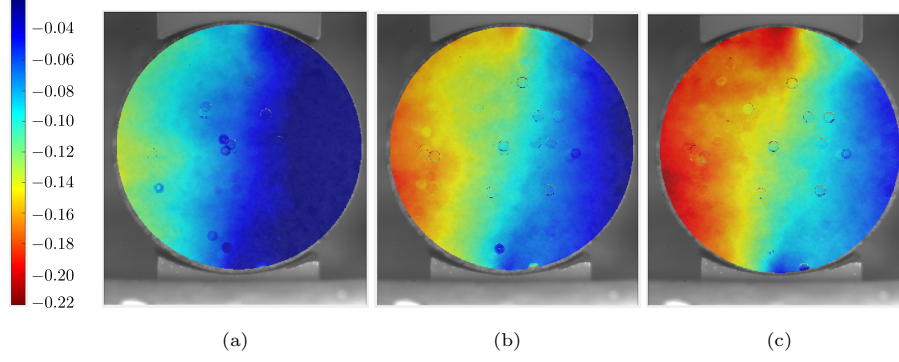


Figure 22: Brazilian tests: Vertical displacement in mm at 1300 °C – DIC sample 6. (a) Point A. (b) Point B. (c) Point C.

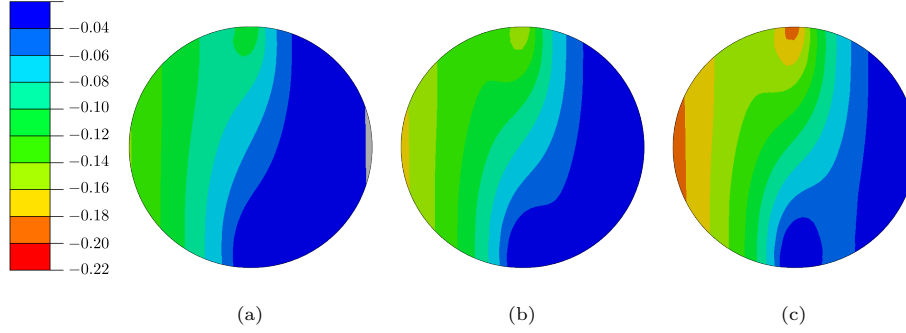


Figure 23: Brazilian tests: Vertical displacement in mm at 1300 °C – Simulation. (a) Point A. (b) Point B. (c) Point C.

566 It is interesting to observe how the vertical displacement of the central line  
 567 of the sample changes when the rigid body motions are included in the simula-  
 568 tion, as shown in Figure 24. When the boundary conditions are symmetric, the  
 569 displacement increases monotonically and equally in both sides of the sample,  
 570 as expected. Nevertheless, a horizontal force in the negative  $x$  direction causes  
 571 an increase in the magnitude of the displacements at the right side of the sam-  
 572 ple (point A in Figure 24), while the left side presents a positive displacement  
 573 during loading (point B in Figure 24). After approximately 1 h of creep defor-

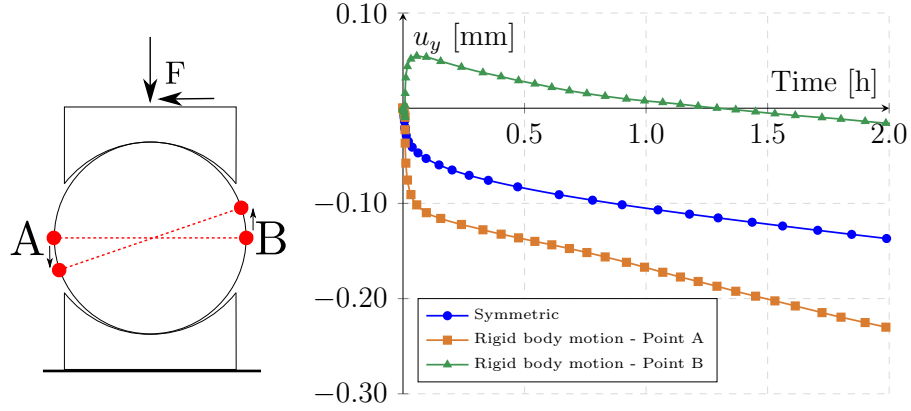


Figure 24: Brazilian tests: Vertical displacement of points A and B.

574 mations, the displacement of the right side becomes negative again, but always  
 575 smaller than at the left. This non-symmetric behavior, [caused by imperfections](#)  
 576 [in the boundary and loading conditions](#), is frequently observed in mechanical  
 577 experiments (de Melo et al., 2020) and has the potential to negatively influence  
 578 the DIC identifications if not properly considered.

## 579 7. Conclusions

580 From previous publications available in the literature, it is clear that refrac-  
 581 tory materials present a different creep behavior under tension and compression.  
 582 Therefore, there is a need to develop a dedicated model, considering that this  
 583 feature is not currently available in the main commercial Finite Element Anal-  
 584 ysis software. In the present work, an asymmetric creep model was proposed,  
 585 that was especially designed to simulate refractory materials at high temper-  
 586 atures. In comparison to previously published works, the proposed model has  
 587 the advantage to be able to represent the primary and secondary creep behavior  
 588 of the material, while keeping the possibility to have independent tests for the  
 589 identification of tension and compression parameters.

590 The main strategy employed in the development of the model was the split  
 591 of the stress tensor into a positive and a negative part, similar to what have

592 been done by Blond et al. (2005), but averaging the contributions of each stress  
593 sign by the equivalent stresses. In this case, the model tends to its symmetric  
594 version when the same material properties are used for both signs.

595 To identify the material parameters of an alumina-spinel material at 1300 °C,  
596 an experimental procedure composed of a Brazilian test in combination with a  
597 subset-based DIC technique was developed, that avoids the problems related  
598 to contact instrumentation at high temperatures. This technique showed to be  
599 successful for the determination of the model parameters.

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